LOGIC, MATHEMATICS AND KNOWLEDGE OF NATURE

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1. THOUGHT AND REALITY

Even a cursory glance at the statements of physics shows that they are obviously of a very diverse character. There are statements such as:

"if a stretched string is plucked, a tone is heard" or

"if a ray of sunlight is passed through a glass prism, then a colored band interspersed with dark lines, is visible on a screen placed behind the prism."

Both of these statements can be tested at any time by observation. We also find statements such as:

"the sun contains hydrogen," and

"the satellite of Sirius has a density of about 60,000" and

"a hydrogen atom consists of a positively charged nucleus around which a negatively charged electron revolves."

These statements cannot by any means be tested by immediate observation but which are made only on the basis of theoretical considerations and likewise are testable only with the help of theoretical considerations. Thus we are confronted by the urgent question: what is the relationship between observation and theory in physics?– and not just in physics, but in science generally. For there is but one science and wherever there is scientific investigation it proceeds ultimately according to the same methods. Only we see everything with the greatest clarity in the case of physics, because it is the most advanced, neatest, most scientific of all the sciences. And in physics, the interaction of observation and theory is especially pronounced, even officially recognized by the establishment of special professorships for *exper-*

imental physics and for *theoretical* physics.

Now the usual view is roughly speaking the following: we have two sources of knowledge by means of which we apprehend "the world," "the reality" in which we are "placed": *experience* or *observation* on the one hand, and thinking on the other. For example, one is engaged either in *experimental* physics or in *theoretical* physics according to one's using the one or the other of these sources of knowledge.

In philosophy we find a long-standing controversy about these two sources of knowledge: Which parts of our knowledge are derived from observation and are "a posteriori," and which are derived from thinking and are "a priori"? Is one of these sources of knowledge superior to the other, and if so, which?

From the very beginning philosophy has raised doubts about the reliability of *observation* (indeed, these doubts are perhaps the source of all philosophy). It is quite understandable why such doubts arose: they spring from the observation that sense-perception is frequently deceptive. At sunrise or at sunset, the snow on distant mountains appears red, but "in reality" it is surely white! A stick which is immersed in water appears crooked, but "in reality" it is surely straight! As a man recedes from me, he appears smaller and smaller to me, but surely he does not change size "in reality"!

Although all the phenomena to which we have been referring have long since been accounted for by physical theories, so that nobody any longer regards them as deceptions caused by sense-perception, the consequences which flow from this primitive, long discarded view still exert a powerful influence. It's been said that if observation is sometimes deceptive, perhaps it is always so! Perhaps everything disclosed by the senses is mere illusion! Everybody knows the phenomenon of dreams, and everybody knows how difficult it is at times to decide whether a given experience was "real life" or "a mere dream." Perhaps, then, whatever we observe is merely a dream! Everybody knows that hallucinations occur, and that they can be so vivid that the per-

son cannot be dissuaded from taking his hallucination for reality. Perhaps then, whatever we observe is only a hallucination! If we look through appropriately polished lenses, everything appears distorted. Who knows, whether perhaps we do not always, unknowingly, look at the world, as it were, through distorting glasses, and therefore see everything distorted, different from what it really is!? This is one of the basic themes of the philosophy of Kant.

But let us return to antiquity. As we said, the ancients believed that they were frequently deceived by observation. But nothing like this ever happened in the case of thought: there were plenty of *delusions of sense*, but no *delusions of thought!* And thus, as confidence in observation was shaken, the belief may have arisen that *thinking* is a method of knowledge which is absolutely superior to observation, indeed the only reliable method of knowledge: whereas observation discloses mere appearance, thought alone grasps true reality.

This "rationalistic" doctrine that thinking is a source of knowledge which is superior to observation, that it is indeed the only reliable source of knowledge, has remained dominant from the climax of Greek philosophy until modern times. I cannot even begin to say what peculiar fruits ripened on this tree of knowledge. Suffice it to say that they proved to have extraordinarily little nourishing value; and thus the "empiricist" reaction, originating in England, slowly gained the upper hand, supported by the tremendous success of modern natural science– the philosophy which teaches that observation is superior to thought, that indeed it is the only source of knowledge: "nothing is in the intellect which was not previously in the senses."

But the empiricist view was soon faced with an insuperable difficulty: how is it to account for the real validity of logical and mathematical statements? Observation discloses to me only the transient, it does not reach beyond the observed; there is no bond that would lead from one observed fact to another– that would compel future observations to have the same result as those

already made. The laws of logic and mathematics, however, claim *absolutely universal* validity: that the door of my room is now closed, I know by observation– next time I observe it, it may be open. That heated bodies expand, I know by observation– yet the very next observation may show that some heated body does not expand. But that two and two make four, holds not only for the case in which I verify by counting– I know with certainty that it holds always and everywhere. Whatever I know by observation could also be otherwise: the door of my room might have been open now, I can easily imagine it; and I can easily imagine that a body does not expand on being heated; but two and two could not occasionally make five. I cannot imagine in any way what it would be like for twice two to equal five.

The conclusion seems inevitable: since the propositions of logic and mathematics have absolutely universal validity, are necessarily certain, and since it must be as they say and cannot be otherwise, these propositions cannot be derived from experience. In view of the tremendous importance of logic and mathematics in the system of our knowledge, empiricism, therefore, seems to be irrevocably refuted. To be sure, in spite of all this, older empiricists have attempted to found logic and mathematics upon experience. According to them we now believe that something must be this way and cannot be otherwise simply because the relevant experience is so old and the relevant observations have been repeated innumerable times. On this view, therefore, it is entirely conceivable that, just as an observation might show that a heated body does not expand, two and two might sometimes make five. This is alleged to have escaped our notice so far because it happens with such extraordinary rarity. Like finding a piece of four-leaved clover which for superstí-

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tious people is a sign of good luck, an occurrence which is not so very rare– how much luckier it would be to come upon a case where two and two make five! One can safely say that upon closer scrutiny these attempts to derive logic and mathematics from experience are fundamentally unsatisfactory, and it is doubtful whether anybody seriously holds this view today.

Rationalism and empiricism having thus, as it were, suffered shipwreck– rationalism, because its fruits lacked nourishing value; empiricism, because it could not do justice to logic and mathematics– *dualistic* views gained the upper hand, with the doctrine that thinking *and* observation are equally legitimate sources of knowledge. Both are indispensable to our comprehension of the world and play a distinctive role in the system of our knowledge. *Thought* grasps the most general laws of all being, as formulated perhaps in logic and mathematics; *observation* provides the detailed filling-in of this framework. On the boundaries between the two sources of knowledge, dualists have held divergent opinions.

For instance, it is disputed whether geometry is *a priori* or *a posteriori*; whether it is based on pure thought or on experience. And the same dispute is encountered in connection with the most fundamental physical laws, e.g. the law of inertia, the laws of the conservation of mass and energy, the law of attraction of masses. All of them have already been acclaimed as *a priori*, as necessities of thought, by various philosophers– but always after they had been established and well confirmed as empirical laws in physics. This was bound to lead to a skeptical attitude. As a matter of fact, there is probably a prevalent tendency among physicists to regard the framework which can be grasped by pure thinking as being as wide and general as possible, and to acknowledge experience as the source of our knowledge of everything that is somehow concrete.

The usual view, then, may be described roughly as follows: from experience we learn certain facts, which we formulate as "laws of nature;" but since by thought we apprehend the most gen-

eral law-like connections in reality (of a logical and mathematical character) that pervade reality, we can control nature on the basis of facts disclosed by observation to a much larger extent than it has actually been observed. For we know in addition that anything which can be deduced from observed facts by application of logic and mathematics must be found to exist. According to this view, the *experimental* physicist provides knowledge of laws of nature by direct observation. The *theoretical* physicist thereafter enlarges this knowledge tremendously by thinking. In this way we are in a position also to assert propositions about processes that occur far from us in space and time, or on account of their magnitude or minuteness, are not directly observable but which are connected with what is directly observed by the most general laws of being, grasped by thought: the laws of logic and mathematics.

This view seems to be strongly supported by numerous discoveries that have been made with the help of theory, such as– to mention just some of the best known– the calculation of the position of the planet Neptune by Leverrier, the calculation of electric waves by Maxwell, the calculation of the bending of light rays in the gravitational field of the sun by Einstein and the calculation of the red-shift in the solar spectrum, also by Einstein.

Nevertheless we are of the opinion that this view is entirely untenable. For on closer analysis it appears that the function of thought is immeasurably more modest than the one ascribed to it by this theory. The idea that thinking is an instrument for learning more about the world than has been observed, for acquiring knowledge of something that has absolute validity always and everywhere in the world, an instrument for grasping general laws of all being, seems to us wholly mystical. Just how should it come to pass that we could predict the neces-

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sary outcome of an observation before having made that observation? Where could our thought acquire the authority to command an observation to come out one way and not another? Why should that which compels our thoughts also compel the course of nature? One would have to believe in some miraculous pre-established harmony between the course of our thinking and the course of nature, an idea which is highly mystical and ultimately theological.

There is no way out of this situation except a return to a purely *empiricist* standpoint, to the view that observation is the only source of knowledge of facts: there is no *a priori* knowledge about matters of fact, *there is no "material" a priori*. However, we must avoid the error committed by earlier empiricists: that of interpreting the propositions of logic and mathematics as mere facts of experience. We must look around for a different interpretation of logic and mathematics.

2. Logic and Reality

Let us begin with logic. The old view of logic is approximately as follows: logic is the account of the most universal properties of things, the account of those properties which are common to all things; just as ornithology is the science of birds, zoology the science of all animals, biology the science of all living things; so logic is the science of *all* things, the science of being as such. If this were the case, it would remain wholly incomprehensible from where logic derives its certainty. For we surely do not know all things. We have not observed everything and hence we cannot know how everything behaves.

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Our thesis, on the other hand is this: logic does not, by any means deal with the totality of things, it does not

deal with objects at all but *only of our way of speaking about objects.* Logic first comes into being through language. The certainty and universal validity of a proposition of logic, or better, its irrefutability, flows precisely from this, that it says nothing about any objects of any kind.

Let us clarify the point by an example. I talk about a well-known plant: I describe it, as is done in botanical reference books, by the number, color, and form of its petals, sepals, and stamens, by the shape of its leaves, stalk, and root, etc., and I make the stipulation: let us call any plant of this kind "apricot," but let us also call it "prunus armeniaca." Thereupon I can pronounce with absolute certainty the universally valid proposition: "every apricot is a prunus armeniaca." It is certainly valid, always and everywhere; it is not refutable by any sort of observation; but it says nothing at all about facts. I learn nothing from it about the plant in question, when it is in bloom, where it may be found, whether it is common or rare. It tells me nothing about the plant; it cannot be disconfirmed by any observation. This is the basis of its certainty and universal validity. The statement merely expresses an agreement we have made about the way we want to talk about the plant in question.

Similar considerations apply to the principles of logic. Let us first consider the two most famous propositions of logic: the law of contradiction and the law of excluded middle. Take, for example, objects which can be assigned a color. We learn, by training, as I am tempted to say, to apply the designation "red" to some of these objects, and we stipulate that the designation "not red" be applied to all other objects. On the basis of this stipulation we now can assert with absolute certainty the proposition that there is no object to which both the designation "red" and the designation "not red" is applied. It is customary to formulate this briefly by saying that: nothing is both "red" and "not red." This is the law of contradiction.

And since we have stipulated that the designation "red" is to be applied to some objects and the designation "not red" to *all*

other objects, we can likewise pronounce with absolute certainty the proposition: everything is either designated as "red" or as "not red," which is usually expressed briefly as follows: Every object is either "red" or "not red." This is the law of excluded middle.

These two propositions, the law of contradiction and the law of the excluded middle, say nothing at all about objects of any kind. They do not tell me if any of them are "red" or "not red," which color they have, or anything else. They merely stipulate a method for applying the designations "red" and "not red" to objects, i.e. they prescribe *a method of speaking about things*. Their universal validity and certainty, their irrefutability, derives precisely from the fact that they say nothing at all about objects.

What applies to these two propositions also applies to the other propositions of logic. We will soon return to this point, but first let us insert another consideration.

We established earlier that there can be no material *a priori,* i.e. no *a priori* knowledge about matters of fact. For we cannot know the outcome of an observation before the observation has taken place. We have made clear to ourselves that no material *a priori* is contained in the laws of contradiction and of excluded middle, since they say nothing about facts. There are those, however, who would perhaps admit that the nature of the laws of logic is as described, yet would insist that there is a material *a priori* elsewhere, e.g. in the statement "nothing is both red and blue" (of course what is meant is: at the same time and place) which is alleged to express real *a priori* knowledge about the nature of things. Even before having made any observation, they say, one can predict with absolute certainty that it will not disclose a thing which is both blue and red; and it is maintained that such *a priori* knowledge is obtained by an intuitive grasp of the essence of colors. If one desires to adhere to our thesis that there is no kind of material *a priori,* one must somehow face statements like "nothing is both blue and red."

I want to attempt this in a few suggestive words, though they cannot by any means do full justice to this problem which is not easy.

It is certainly true that, even before we have made an observation, we can say with complete certainty that our observation will not show that an object is both red and not red, or that an *apricot* is not a *prunus armeniaca*. The first statement, however, is not a case of a material *a priori* any more than the second. Like the statements "every apricot is a prunus armeniaca": and "nothing is both red and not red," the statement "nothing is both blue and red" says nothing at all about the nature of things; it likewise refers only to our proposed manner of speaking about objects, of applying designations to them.

Earlier we said: there are some objects that we call "red," every other object we call "not red," and from this we derive the laws of contradiction and excluded middle. Now we say: some objects we call "red," some *other* objects we call "blue," and *other* objects again we call "green," etc. But if it is in this way that we ascribe color designations to objects, then we can say with certainty in advance: in this procedure no object is designated both as "red" and as "blue," or more briefly: no object is both red and blue. The reason why we can say this with certainty is that we have mutually stipulated that this is how we assign colors to objects.

We see, then, that there are two totally different kinds of statements: those which really say something about objects, and those which do not say anything about objects but only stipulate rules for speaking about objects. If I ask "what is the color of Anie's new dress?" and get the answer "Anie's new dress is *not* both red and blue (all over)," then no information about this dress has been given to me at all. I have been made no wiser by it. But if I get the answer "Anie's new dress is red," then I have received some genuine information about the dress. Let us clarify this distinction in terms of one more example.

A statement which really says something about the objects which it mentions, is the following: "if you heat this piece of iron up to 800°, it will turn red; if you heat it up to 1300°, it will turn white." What makes the difference between this statement and the statements cited above, which say nothing about facts? The assignment of temperature designations to objects is *independent* of the assignment of color designations, whereas the color designations "red" and "not red," or "red" and "blue" are applied to objects in *mutual dependence*. The statements "Anie's new dress is either red or not red" and "Anie's new dress is not both red and blue" merely express this kind of dependence. Therefore they do not make any assertion about that dress, and are for that reason absolutely certain and irrefutable. The above statement about the piece of iron, on the other hand, relates independently given designations, and therefore really says something about that piece of iron and is for just that reason is uncertain and refutable by observation.

The following example may make the difference between these two kinds of statements particularly clear. If someone tells me: "I raised the temperature of this piece of iron to 800° but it did not turn red," then I would test his assertion. The result of the test may be that he was lying, or that he was the victim of an illusion, but perhaps it would turn out that– contrary to my previous beliefs– there are cases where a piece of iron heated to 800° does not become red-hot. In that case I would just change my opinion about the reaction of iron to heating. But if someone tells me "I raised the temperature of this piece of iron to 800°, and this made it turn both red and not red" or "it became both red and white," then I will certainly make no test whatever. Nor will I say "he has told me a lie," or "he has become the victim of an illusion," and it is quite certain that I would not change my beliefs about the reaction of iron to heating. The point is– it is best to express it in language which any card player is familiar with– that the man has revoked: he has violated the rules in accordance with which we want to speak, and I shall refuse to speak with him any

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longer. It is as though one attempted in a game of chess to move the bishop orthogonally rather than diagonally. In this case too, I would not make any tests, I would not change my beliefs about the behavior of things, but I would refuse to play chess with him any longer.

To sum up, we must distinguish between two kinds of statements: those which say something about facts and those which merely express the way in which the rules which govern the application of words to facts depend upon each other. Let us call statements of the latter kind *tautologies:* they say nothing about objects and are for this very reason certain, universally valid and irrefutable by observation. Whereas the statements of the former kind are not certain and are refutable by observation. The logical laws of contradiction and of the excluded middle are tautologies, likewise, e.g., the statement "nothing is both red and blue."

And now we assert that in the same way, all the other laws of logic are tautologies. Let us, therefore, return to logic once more in order to clarify the matter by an example. As we said, the designation "red" is applied to certain objects and the convention is adopted of applying the designation "not red" to any other object. It is this agreement about the use of negation which is expressed by the laws of contradiction and of the excluded middle. Now we add the convention– still taking our examples from the domain of colors– that any object which is called "red" is also to be called "red or blue," "blue or red," "red or yellow," "yellow or red," etc.; that every object which is called "blue," is also called "blue or red," "red or blue," "blue or yellow," "yellow or blue," etc., and so on. On the basis of this convention, we can again assert with complete certainty the proposition: "every red object is either red or

blue." This is again a tautology. We do not speak about the objects, but only about our manner of talking about them.

If once more we remind ourselves of the way in which the designations "red," "not red," "blue," "red or blue," etc. are applied to objects, we can more-over assert with complete certainty and irrefutability: everything to which both designations "red or blue" and "not red" are applied, is also designated as "blue"– which is usually put more briefly– if a thing is red or blue and not red, then it is blue. Which is again a tautology. No information about the nature of things is contained in it; it only expresses the sense in which we use the logical words "not" and "or."

... logical deduction ... makes us aware of all that we have implicitly asserted

This brings us to a very fundamental point. The agreement about the use of the words "not" and "or" is such that in asserting the two propositions "object A is either red or blue" and "object A is not red," I have implicitly already asserted "object A is blue." This is the essence of *logical deduction.* It is not in any way based on real connections between states of affairs, which we apprehend in thought. On the contrary, it has nothing at all to do with the nature of things, but derives from our manner of speaking about things. If a person refused to recognize logical deduction, he would not thereby manifest a different belief from mine about the behavior of things, but he would refuse to speak about things according to the same rules as I do. I could not convince him, but I would have to refuse to speak with him any longer, just as I would refuse to play chess with a partner who insisted on moving the bishop orthogonally.

What logical deduction accomplishes, then, is this: it makes us aware of all that we have implicitly asserted– on the basis of agreements regarding the use of language– in asserting a system

of propositions, just as, in the above example, "object A is blue" is implicitly asserted by the assertion of the two propositions "object A is red or blue" and "object A is not red." In saying this we have already suggested the answer to the question, which naturally must have forced itself on the mind of every reader who has followed our argument: if it is really the case that the propositions of logic are tautologies, that they say nothing about objects, what purpose does logic serve?

... logical deductions ... have significance for us because we are not omniscient

The logical propositions which were used as illustrations were derived from our agreements about the use of the words "not" and "or" (and it can be shown that the same holds for all the propositions of so-called propositional logic). Let us, then, first ask for what purpose the words "not" and "or" are introduced into language. **The reason would seem to be that we are not omniscient.** If I am asked about the color of the dress worn by Anie yesterday, I may not be able to remember its color. I cannot say whether it was red or blue or green; but perhaps I will be able to say at least "it was not yellow." Were I omniscient, I would know its color. There would be no need to say "it was not yellow"; I could say "it was red." Or again: my neice has written to me that she received a cocker-spaniel as a present. As I have not seen it yet, I do not know its color; I cannot say "it is black" nor "it is brown"; but I *am* able to say "it is black or brown." Were I omniscient, I could do without this "or" and could say immediately "it is brown."

Thus logical propositions, though being purely tautologous; and logical deductions, though being nothing but tautological transformations, have significance for us because we are not

omniscient. Our language is so constituted that in asserting certain propositions, we implicitly assert other propositions– but we do not see immediately all that we have implicitly asserted in this manner. It is only logical deduction that makes us conscious of it. I assert, e.g., the propositions "the flower which Gayle wears in her hair is either a rose or a carnation," "if Gayle wears a carnation in her hair, then it is white," "the flower which Gayle wears in her hair is not white." Perhaps I am not consciously aware that I have implicitly asserted also "the flower which Gayle wears in her hair is a rose"; but logical deduction brings it to my consciousness. To be sure, this does not mean that I know whether the flower which Gayle wears in her hair really is a rose. If I notice that it is not a rose, then I must not maintain my previous assertions– otherwise I sin against the rules of speaking, I revoke.

3. Mathematics and Reality

If I have succeeded in clarifying somewhat the role of logic, I may now be quite brief about the role of *mathematics*. The propositions of mathematics are of exactly the same kind as the propositions of logic: they are tautologous, they say nothing at all about the objects we want to talk about, but concern only the manner in which we want to speak about them.

The reason why we can assert with certainty and universal validity the proposition: 2+3=5; why we can say, even before any observations have been made, and can say it with complete certainty, that it will not turn out that 2+3=7, is that by "2+3" we mean the same as by "5"– just as we mean the same by "*prunus armeniaca*" as by "apricot." For this reason, no botanical investigation, however subtle, could disclose that an instance of the species "apricot" is not a *prunus armeniaca*.

We become aware of meaning the same by "2+3" and by "5," by going back to the meanings of "2" "3" "5" & "+" and making tautological transformations until we just see that "2+3" means the same as "5." It is such successive tautological transformation that

is meant by "calculating." The operations of addition and multiplication which are learnt in school are directives for such tautological transformation. Every mathematical proof is a succession of such tautological transformations. Their utility, again, is due to the fact that, for example, we do not by any means see immediately that we mean by "24x31" the same as by "744." But if we calculate the product "24x31," then we transform it step by step, in such a way that in each individual transformation we recognize that on the basis of the conventions regarding the use of the signs involved (in this case numerals and the signs "+" and "x") what we mean after the transformation, is still the same as what we meant before it. Finally we become consciously aware of meaning the same by "744" as by "24x31." To be sure, the proof of the tautological character of mathematics is not yet complete in all details. This is a difficult and arduous task; yet we have no doubt that the belief in the tautological character of mathematics is essentially correct.

it is difficult to believe that the whole of mathematics ... should admit of being resolved into tautologies

There has been prolonged opposition to the interpretation of mathematical statements as tautologies. Kant contested the tautological character of mathematics emphatically, and the great mathematician Henri Poincaré, to whom we are greatly indebted also for philosophical criticism, went so far as to argue that since mathematics cannot possibly be a huge tautology, it must somewhere contain an *a priori* principle. Indeed, at first glance it is difficult to believe that the whole of mathematics, with its theorems that cost such labor to establish, with its results that so often surprise us, should admit of being resolved into tautologies. But there is just one little point which this argument overlooks: it overlooks the fact that **we are not omniscient**.

An omniscient being, indeed, would at once know everything that is implicitly contained in the assertion of a few propositions. He would know immediately that on the basis of the conventions concerning the use of the numerals and the multiplication sign, "24x31" is synonymous with "744." **An omniscient being has no need for logic and mathematics.** We on the other hand, first have to make ourselves conscious of this by successive tautological transformations, and hence it may prove quite surprising to us that in asserting a few propositions we have implicitly also asserted a proposition which seemingly is entirely different from them, or that we mean the same by two complexes of symbols which are externally altogether different.

An omniscient being has no need for logic and mathematics

4. Theory and Experience

Let us now make it clear that our view is poles apart from the older – or as it might perhaps be called, Platonistic view– that the world is constructed according to the laws of logic and mathematics ("God is always doing mathematics"); that our thought, a feeble reflection of God's omniscience, is an instrument given to us for comprehending the eternal laws of the world. No! Our thinking cannot give insight into any sort of reality. It cannot bring us information of any fact in the world. It only refers to the manner in which we speak about the world. All it can do is to transform tautologically what has been said. There is no possibility of piercing through the sensible world disclosed by observation to a "world of true being." Any metaphysics is impossible! Impossible, not because the task is too difficult for our human thinking, but because it is meaningless. Every attempt to do metaphysics is an attempt to speak in a way that contravenes the

agreement as to how we wish to speak, comparable to the attempt to capture the queen (in a game of chess) by means of an orthogonal move of the bishop.

Let us now return to the problem with which we started: what is the relationship between observation and theory in physics? We said that the usual view was roughly this: experience teaches us the validity of certain laws of nature, and since our thinking gives us insight into the most general laws of all being, we know that likewise anything which is deducible from these laws of nature by means of logical and mathematical reasoning must be found to exist. We see now that this view is untenable; for thinking does not grasp any sort of laws of being. Never and nowhere, then, can thought supply us with knowledge about facts that goes beyond the observed. But what, then, should we say about the discoveries made by means of theory on which, as we pointed out, the usual view so strongly relies for its support? Let us ask ourselves, e.g., what was involved in the computation of the position of the planet Neptune by Leverrier?

Newton noticed that the familiar motions, celestial as well as terrestrial, can be well described in a unified way by the assumption that between any two mass points a force of attraction is exerted which is proportional to their masses and inversely proportional to the square of their distance. And it is because this assumption enables us to give a satisfactory description of the familiar motions, that he *made* it, i.e. he asserted tentatively, as an hypothesis, the law of gravitation. Between any two mass points, there is a force of attraction which is proportional to their masses and inversely proportional to the square of their distance.

He could not pronounce this law as a *certainty*, but only as an hypothesis. For nobody can know that such is really the behavior of every pair of mass points; nobody can observe all mass points. But having asserted the law of gravitation, one has implicitly asserted many other propositions; that is, all propositions which are deducible from the law of gravitation (together with

data immediately derivable from observation) by calculation and logical inference.

It is the task of theoretical physicists and astronomers to make us conscious of everything we implicitly assert along with the law of gravitation. And Leverrier's calculations made people aware that the assertion of the law of gravitation implies that at a definite time and definite place in the heavens a hitherto unknown planet must be visible. People looked and actually saw that new planet– the hypothesis of the law of gravitation was confirmed. But it was not Leverrier's calculation that proved that this planet existed, but the looking, the observation. This observation could just as well have had a different result. It could just as well have happened that nothing was visible at the computed place in the heavens– in which case the law of gravitation would not have been confirmed and one would have begun to doubt whether it is really a suitable hypothesis for the description of the observable motions.

Indeed, this is what actually happened later: in asserting the law of gravitation, one implicitly asserts that at a certain time the planet Mercury must be visible at a certain place in the heavens. Whether it would actually be visible at that time and at that place, only observation could disclose. But observations showed that it was not visible at exactly the required position in the heavens. And what happened? They said: since in asserting the law of gravitation we implicitly assert propositions which are not accurate, we cannot maintain the hypothesis of the law of gravitation. Newton's theory of gravitation was replaced by Einstein's theory.

It is not the case, then, that we know through experience that certain laws of nature are valid, and– since by our thinking we grasp the most general laws of all being– therefore also know that whatever is deducible from these laws by reasoning must exist. On the contrary, the situation is this: there is not a single law of nature which we know to be valid; the laws of nature are *hypotheses* which we assert tentatively. But in asserting such

laws of nature, we implicitly assert also many other propositions, and it is the task of thinking to make us conscious of the implicitly asserted propositions. As long as these implicitly asserted propositions, to the extent that they are about the directly observable, are confirmed by observation, these laws of nature are confirmed and we adhere to them; but if these implicitly asserted propositions are not confirmed by observation, then the laws of nature have not been confirmed and are replaced by others.

From *Logik, Mathematik und Naturerkennen* Edited and updated.